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M. ASLAM

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Technical Report TI - NAVY - 1

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PREDICTION OF CUMULATIVE FATIGUE DAMAGE

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ABSTRACT

In this report, fatigue in general and some prominent theories concerning prediction of cumulative fatigue damage are discussed. A computer program was developed to calculate the cumulative fatigue damage and fatigue life using the predictive equation developed by I. R. Kramer (8). Test results generated by Kramer for 2014-T6 aluminum alloy were used to determine cumulative fatigue damage and fatigue life. The experimental values of fatigue damage and life are found to be in agreement with those predicted.



Accession For

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PREDICTION OF CUMULATIVE FATIGUE DAMAGE

Failure of machine members under repeated or fluctuating stresses is called fatigue failure. This type of failure occurs below the ultimate strength of the material and quite often even below the yield strength. Failure begins with a crack at the surface. The initial crack is so small and minute that it cannot be detected by the naked eye and is even difficult to locate in X-ray inspection. The crack develops at a point of discontinuity such as a keyway, a hole, an inspection or stamp mark, an internal crack, or some irregularities caused by machining. Once a crack has developed, the stress concentration effect becomes greater and the crack propagates more rapidly. As the stress area decreases in size, the stress increases in magnitude until, finally, the remaining area fails suddenly. A fatigue failure is, therefore, characterized by two distinct areas of failure. The first is due to the progressive development of crack, while the second is due to sudden failure which resembles the fracture of brittle materials. Many static failures are visible and give warning in advance. But a fatigue failure gives no warning, it is sudden and total, and hence dangerous.

Fatigue Strength and Endurance Limit

To establish the fatigue strength of a material, quite a number of tests are necessary because of the statistical nature of fatigue. The

first test is made at a stress which is somewhat under the ultimate strength of the material. The second test is made with a stress which is less than that used in the first. This process is continued and the results are plotted as an S-N diagram (Fig. 1). This chart may be plotted on semi-log or on log-log paper. In the case of ferrous metals and alloys, the graph becomes horizontal after the material has been stressed for a certain number of cycles. The ordinate of the S-N diagram is called the Fatigue Strength, corresponding to the number of cycles N required to produce failure. When the curve becomes horizontal, as it does for steel, failure will not occur if the stress is below this level, no matter how many stress cycles are applied. This fatigue strength is called the Fatigue Limit or Endurance Limit. Different stress components used in fatigue analysis are shown in Figure 2.

Low-Cycle and High-Cycle Fatigue

A complete S-N curve may be divided into two portions: the low-cycle range and the high-cycle range. There is no dividing line between the two. The investigators arbitrarily say that up to about 10^3 or 10^4 is low cycle and beyond 10^4 cycles is high cycle. The low-cycle fatigue is important in pressurized fuselages, missiles, space ship launching equipment, etc. The failure mechanism in the low-cycle range is close to that in static loading, but the failure mechanism in the high-cycle range is different and may be termed "true fatigue."

Cumulative Fatigue Damage

Fatigue loads applied to machine parts and structures are seldom of constant magnitude. Machines have to be started up and

stopped, overloads occur, and transient vibrations of the part may impose high frequency stresses.

In Figure 3 assume that a specimen is subjected to an alternating stress of amplitude σ_A for N_A cycles. The fatigue strength σ_A corresponds to N_B cycles of life, and the remaining life at this same stress magnitude is $N_B - N_A$ cycles. Consequently, the specimen has accumulated some fatigue damage at this stress magnitude. OBD represent the S-N diagram of the virgin specimen. $N_C = N_B - N_A$ is the remaining useful life of the specimen. Now locate point C, and construct line OCE, which is the new S-N diagram having a lower endurance limit σ'_e . The damage done by overstressing is, therefore, the difference in the endurance limits ($\sigma_e - \sigma'_e$).

Equivalent Cycle Approach. Let us consider the two-level sinusoidal stress history shown in Figure 4 with maximum stress levels S_1 and S_2 applied to a material alternately in groups of \bar{n}_1 and \bar{n}_2 cycles, respectively. The number of cycles to failure at stress conditions S_1 and S_2 are represented by N_1 and N_2 respectively. From Figure 5:

$$\bar{D}_1 = \frac{\bar{n}_1}{N_1} = \frac{\bar{n}_{21}}{N_2}$$

or

$$\bar{n}_{21} = \frac{N_2}{N_1} \bar{n}_1$$

\bar{n}_{21} is the number of cycles applied at stress condition S_2 that would produce the same amount of damage as \bar{n}_1 cycles applied at stress condition S_1 .

By a similar analysis

$$\bar{n}_{12} = \frac{N_1}{N_2} \bar{n}_2$$

The number of cycles \bar{n}_{2B} at stress condition S_2 that yields the same amount of damage as that caused by the block containing \bar{n}_1 cycles at stress condition S_1 , plus \bar{n}_2 cycles at stress condition S_2 .

$$\bar{n}_{2B} = \bar{n}_{21} + \bar{n}_2$$

$$\bar{n}_{2B} = N_2 \left(\frac{\bar{n}_1}{N_1} + \frac{\bar{n}_2}{N_2} \right)$$

The number of repetitive blocks to failure n_{Bf} is

$$n_{Bf} = \frac{N_2}{\bar{n}_{2B}} \Rightarrow n_{Bf} \cdot \bar{n}_{2B} = N_2$$

Replace \bar{n}_{2B}

$$n_{BF} \cdot N_2 \left(\frac{\bar{n}_1}{N_1} + \frac{\bar{n}_2}{N_2} \right) = N_2 \Rightarrow n_{BF} \left(\frac{\bar{n}_1}{N_1} + \frac{\bar{n}_2}{N_2} \right) = 1$$

or

$$\sum_{q=1}^2 \frac{n_{qF}}{N_q} = 1$$

Equivalent Stress Approach. The equivalent stress approach derives its name from the consideration that there exists a stress condition which will cause failure in the same total number of cycles

as that needed by the complex history. Total damage at failure D_F , associated with a multilevel h distinct sinusoidal group is given by

$$D_F = \sum_{q=1}^h \frac{n_{qF}}{N_q}$$

where n_{qF} is the number of cycles in the failure history at stress condition S_q . The total number of cycles required to cause failure is N_F . Equivalent stress approach specifies that a stress condition S_e would also produce failure in N_F cycles. If D_{F_e} denotes the damage at failure associated with the stress condition S_e then $D_F = D_{F_e}$.

If N_e is the number of cycles to failure at stress condition S_e then

$$N_e = N_F \Rightarrow \frac{N_F}{N_e} = 1 = D_{F_e}.$$

THEORIES ABOUT THE PREDICTION OF CUMULATIVE FATIGUE DAMAGE

Miner's Theory

The simplest and most often used is the theory proposed by Miner. This theory is referred to as the linear cumulative damage rule and utilizes the simple cycle ratio as its basic measure of damage. If a multilevel sinusoidal stress history is applied to a structural material, it is hypothesized in Miner's Theory that:

- a) each group of sinusoids contributes an amount of damage given by the linear cycle ratio for the group.
- b) the damage done by any group of sinusoids is not dependent on the group's location in the stress history.
- c) the total applied damage is equal to the sum of the damages contributed by each sinusoidal group.

$$\bar{D}_q = \frac{\bar{n}_q}{N_q}$$

\bar{D}_q = The damage resulting from this group of sinusoids,

\bar{n}_q = Number of sinusoids in this group, and

N_q = Number of sinusoids to produce failure at maximum stress level S_q .

If D_B is damage produced by the block of h distinct sinusoidal groups then

$$D_B = \sum_{q=1}^h \bar{D}_q = \sum_{q=1}^h \frac{\bar{n}_q}{N_q}$$

If there are n_B number of the basic block of h sinusoidal group, the total damage D is given by

$$D = n_B D_B = n_B \sum_{q=1}^h \frac{\bar{n}_q}{N_q} = \sum_{q=1}^h \frac{n_q}{N_q}$$

For failure to occur D must equal unity.

It has been found from testing under multilevel sinusoidal histories that Miner's theory predicts a longer life than that actually witnessed.

Grover's Theory

Grover's theory considers that the fatigue life of a material subjected to a complex stress history is composed of two stages: a) an initial number of cycles, N_F' required to nucleate the crack and b) N_F'' number of cycles needed to propagate this crack to the failure of the material. The total number of cycles N_F required to cause the failure of the material is thus given by

$$N_F = N_F' + N_F''$$

Now we consider a multilevel sinusoidal failure history with h distinct sinusoidal stress conditions

$$N_F' = \sum_{q=1}^h n_{qF}' \quad \text{and} \quad N_F'' = \sum_{q=1}^h n_{qF}''$$

where n'_{qF} and n''_{qF} designate respectively the number of cycles at stress condition S_q applied during the crack nucleation and crack propagation. If N'_q is the number of cycles in the failure producing crack and N''_q is the number of cycles needed to propagate the crack to failure at stress S_q and N_q is the number of cycles to failure at S_q then

$$N_q = N'_q + N''_q$$

or

$$\frac{N'_q}{N_q} + \frac{N''_q}{N_q} = 1$$

Grovers theory now utilizes Miner's theory separately for the nucleation stage and for the propagation to failure stage.

$$\sum \frac{n'_{qF}}{N_q} = 1$$

and

$$\sum \frac{n''_{qF}}{N''_q} = 1$$

This theory has a serious setback of inability to find exactly the number of cycles required to nucleate the crack. This theory is unconservative like Miner's Theory.

Marco-Starkey Theory

The Marco-Starkey specification for the damage D arising from n cycles applied at stress condition S with an associated number of cycles to failure N is given by:

$$D = (n/N)^x$$

The exponent x is a variable quantity whose magnitude is dependent on the applied stress condition. Marco and Starkey consider that x has a magnitude greater than unity and approaches unity as the stress condition becomes severe as shown in Figure 6. This theory is conservative and has limited use because of the difficulty to determine the exponent x , and its dependency on stress under complex cyclic conditions.

Shanley's Theory

This theory uses equivalent stress approach. The damage is given by

$$D = C S^{kb} n$$

n = number of cycles applied at stress condition S .

b = the slope of the central portion of the S - N diagram.

C & k = material constants where k is greater than 1.

D is a function of number of applied cycles rather than the cyclic ratio n/N as in previous cases.

It is seen that the equation for the central portion of the S - N diagram can be put in the form

$$N = \frac{1}{C S^b},$$

therefore,

$$C = \frac{1}{N_r S_r^b}$$

N_r and S_r are reference number of cycles and stress respectively. If we take $k = 1$ then $D = n/N$, which is Miner's theory. The value of $k = 2$

is mostly used for Shanley's theory. This theory yields shorter fatigue life than that predicted by Miner.

Corten-Dolan Theory

For the Corten-Dolan theory the damage D in a material due to n cycles of pure sinusoidal stress history can be expressed as:

$$D = r n^a \quad (1)$$

r = function of stress condition

a = material constant

If $a = 1$ and $r = C S^{kb}$ then Corten-Dolan theory yields to Shanley's theory. Corten and Dolan used an equivalent cycle approach for determining a fatigue failure criterion based on the damage specification in Eq. (1).

Freudenthal-Heller Theory

Freudenthal and Heller proposed the modification to the central portion of the S-N diagram. In terms of modified number of cycles to failure N_{qm} for stress condition S_q can be expressed as

$$\frac{N_{qm}}{N_r^*} = \left(\frac{S_r^*}{S_q} \right)^\delta \quad (2)$$

where S_r^* = reference stress condition that is unrelated to any applied stress history,

N_r^* = associated number of cycles to failure, and

δ = slope of modified S-N diagram.

Expression for the conventional S-N diagram

$$\frac{N_q}{N_r^*} = \left(\frac{S_r^*}{S_q} \right)^b \quad (3)$$

The above equations (2 & 3) indicate that the modified and conventional S-N diagrams will be coincident only at stress condition $S_q = S_r^*$ for which $N_q = N_r^*$. The reference stress condition S_r^* will usually have such intensity that N_r^* will fall in the range of 10^3 to 10^4 cycles. Total number of cycles N_{Fm} required to cause failure of a material under a multilevel sinusoidal stress history is as follows, based on the modified S-N diagram shown in Figure 7.

$$N_{Fm} = \left(\sum_{q=1}^h \frac{\alpha_q}{N_{qm}} \right)^{-1}$$

The total number of cycles N_F required to cause failure, based on the conventional S-N diagram, is expressed as

$$\frac{N_{Fm}}{N_F} = \frac{\left[\sum_{q=1}^h \frac{\alpha_q}{N_{qm}} \right]}{\left[\sum_{q=1}^h \frac{\alpha_q}{N_{qm}} \right]}$$

$$N_{Fm} = N_F \frac{\sum_{q=1}^h \alpha_q (S_q/S_r^*)^b}{\sum_{q=1}^h \alpha_q (S_q/S_r^*)^\delta}$$

Kramer's Theory

Kramer conducted some experiments and proposed that while materials are subjected to fatigue cycles, the work hardening of

surface layer takes place and consequently the proportional limit for the material is increased with increased number of cycles. He defined this increase in proportional limit as the surface layer strength (σ_s). He further investigated that when this surface layer stress reaches a critical value (σ_s^*) the failure producing crack is propagated. He showed that σ_s is independent of the stress magnitude applied.

N_0 = number of cycles to initiate the propagating crack

N_F = number of cycles to failure

$$\frac{N_0}{N_F} = \text{Const} = 0.7 \text{ for aluminum}$$

$$S = d_s/dN$$

$$\sigma_s = SN$$

$$\text{or } \sigma_s^* = S N_0$$

$$D = \frac{\sigma_s}{\sigma_s^*} \quad \text{a crack will initiate when } \sum \frac{\sigma_{s_i}}{\sigma_s^*} = 1 \text{ or } S_i N_i = \sigma_s^*$$

The incremental rate of change of surface stress σ_s at the first stress level is given by:

$$S_1 = \alpha_\sigma P_1$$

After N_1 cycles, the maximum stress is increased to σ_2 and the incremental rate of change of surface stress at this second level will be modified as

$$S_{II} = \left(\frac{S_1}{S_2}\right)^{f_1} \sigma_2^p$$

$$S_{II} = \left(\frac{\sigma_1}{\sigma_2}\right)^{Pf_1} \sigma_2^P$$

Similarly at third stress level

$$S_{III} = \left(\frac{S_{II}}{\sigma_3}\right)^{f_2} \sigma_3^P$$

Replace S_{II}

$$S_{III} = \left(\frac{\sigma_1}{\sigma_2}\right)^{Pf_1} \left(\frac{\sigma_2^P}{\sigma_3^P}\right)^{f_2} \sigma_3^P$$

$$S_{III} = \left(\frac{\sigma_1}{\sigma_2}\right)^{Pf_1 f_2} \left(\frac{\sigma_1}{\sigma_2}\right)^{Pf_2} \sigma_3^P$$

And so on.

$$S_i N_i = \sigma_s^*$$

$$\sigma_1^P N_1 + \sigma_2^P N_2 \left(\frac{\sigma_1}{\sigma_2}\right)^{Pf_1} + \sigma_3^P N_3 \left(\frac{\sigma_1}{\sigma_2}\right)^{Pf_1 f_2} \left(\frac{\sigma_2}{\sigma_3}\right)^{Pf_2} + \dots = \frac{\sigma_s^*}{\alpha} = \beta$$

$$\frac{\sigma_1^P N_1}{\beta} + \frac{\sigma_2^P N_2}{\beta} \left(\frac{\sigma_1}{\sigma_2}\right)^{Pf_1} + \frac{\sigma_3^P N_3}{\beta} \left(\frac{\sigma_1}{\sigma_2}\right)^{Pf_1 f_2} \left(\frac{\sigma_2}{\sigma_3}\right)^{Pf_2} + \dots = 1$$

N_1, N_2, N_3 --number of applied cycles

$\sigma_1, \sigma_2, \sigma_3$ --applied stresses

f_1, f_2, f_2 --previous history damage terms

$P = -1/m$ --m is slope of S-N curve

$$\beta = C^P$$

C = material constant

$$m = \frac{\log \sigma_1 - \log \sigma_2}{\log N_1 - \log N_2}$$

$$\log \sigma = m \log N + \log C$$

$$\Rightarrow C$$

$$\log \sigma = \log N^m + \log C$$

$$\log \sigma = \log C N^m$$

or

$$\sigma = C N^m \Rightarrow N^m = \frac{\sigma}{C}$$

$$N = \sigma^{1/m} \cdot C^{-1/m}$$

$$N = \sigma^{-P} C^P$$

or

$$N = \beta \sigma^{-P}$$

Kramer's equation uses only the S-N diagram and also takes care of the previous damage histories in the following terms.

Experimental Work. The fatigue specimens used to measure the surface layer stress and to determine the effects of removing the surface layer on fatigue life were machined from 15mm diameter rods of 2014-T6 aluminum. These specimens had a diameter of 0.16 and a gage length of 0.30 in. Before testing, all specimens were electropolished to remove about 0.004 in to obtain the same surface finish. The fatigue tests were conducted in an electrohydraulic machine in tension-compression.

The change in the surface layer stress at various applied stress amplitudes was determined by measuring the increase in the proportional limit as a function of the number of cycles. However, when the surface layer was removed after cycling, the proportional limit decreased to the same value as that of the uncycled specimen. It follows from these fatigue tests that the work hardening of the specimens during cycling is confined primarily to the surface layer. The increase in the proportional limit is then equal to the strength of the surface layer.

For the determination of the change in the proportional limit, the stress-strain measurements were made immediately after cycling to minimize surface layer losses due to relaxation effects. An extensometer with gage length of 0.30 in was attached to the specimen after the cycling sequence and measurements were begun in less than one minute.

Analysis. To verify the validity of his equation, Kramer fatigued seven specimens, using four stages of stress with different combinations of number of cycles employed. He used four different stress sequences, i.e., low to high, low to high-mixed, high to low, and high to low-mixed.

In Table 1, specimen numbers 1 through 3 are stressed, using low to high stress pattern, numbers 4 and 5 are stressed high to low, number 6 is tested low to high-mixed and number 7 is tested following high to low-mixed stress sequence. Table 2 compares the total cumulative fatigue damage calculated using Kramer's and Miner's equations. Table 3 shows actual number of cycles taken by specimens in the final (3rd or 4th) stage and the number of cycles predicted using Kramer's equation in the final stage.

CONCLUSIONS

1. The predicted life is in good agreement with that determined experimentally for reversed stress conditions.
2. The material constants P and β are calculated from S-N diagram. The accuracy in predicting the fatigue life under cumulative damage depends to a large degree on P and β . P and β vary tremendously with the slope of S-N diagram. Therefore, the accurate generation of S-N diagram is the most important factor for obtaining good results using Kramer's equation.
3. The cumulative damage values calculated by Kramer and computed at Tuskegee Institute differ a little bit. Most probably explanation could be that Kramer rounded off the experimental stress values.

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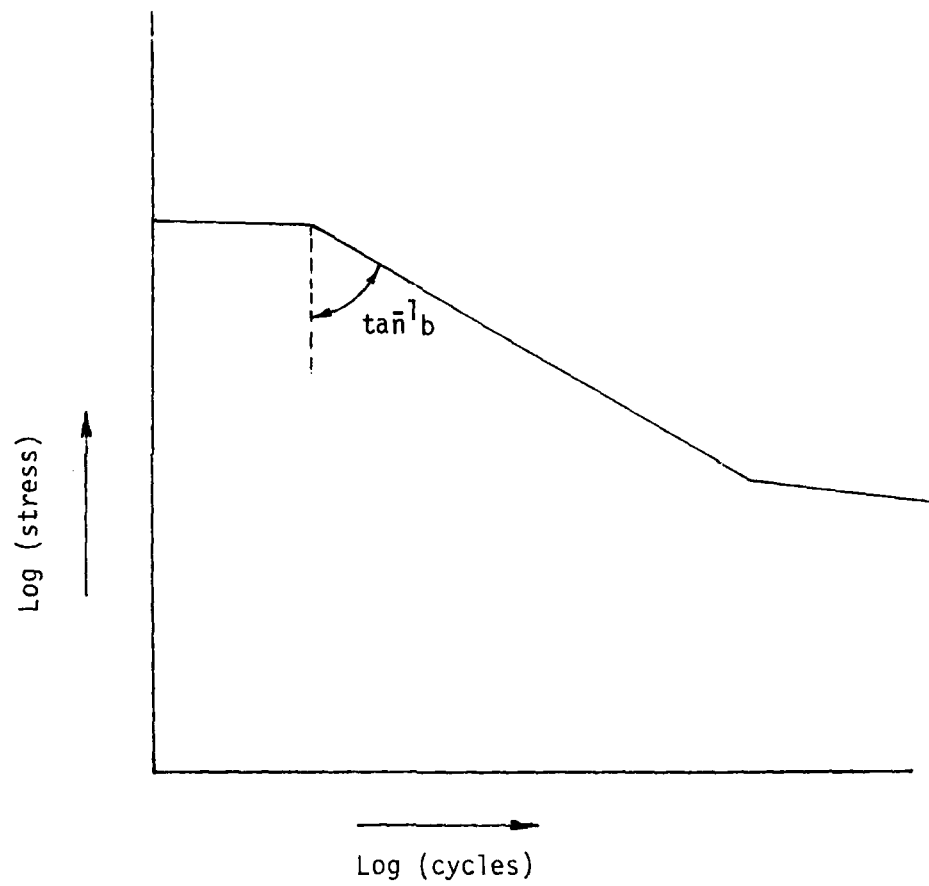
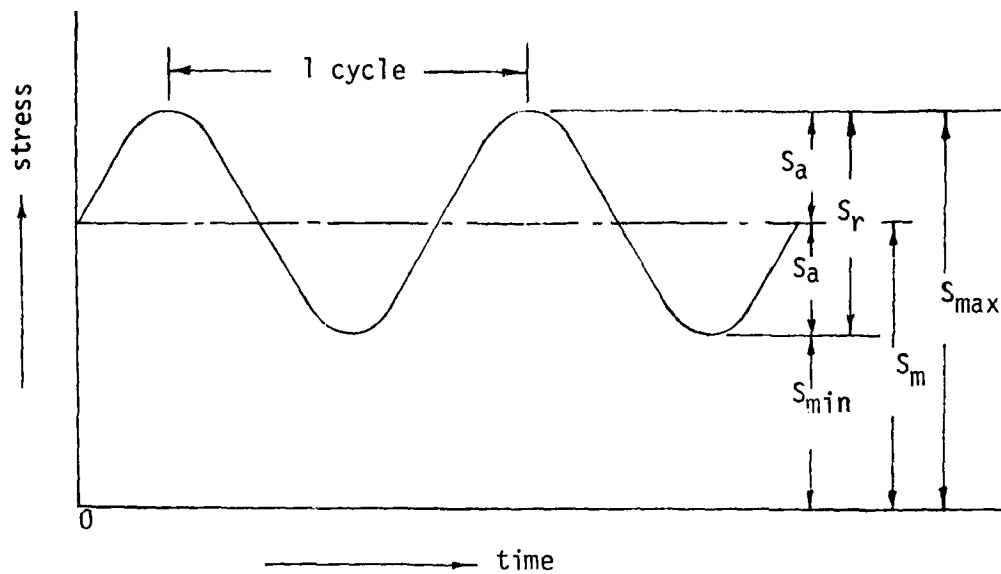


Figure 1. S-N Diagram



The stress components shown in fig. 2 are as follows:

S_{min} = Minimum Stress

S_{max} = Maximum Stress

S_a = Stress amplitude

S_r = Stress Range = $S_{max} - S_{min} = 2S_a$

S_m = Mean stress = $\frac{S_{max} + S_{min}}{2}$

R = Stress Ratio = S_{min} / S_{max} .

Figure 2. Fatigue Stress Components

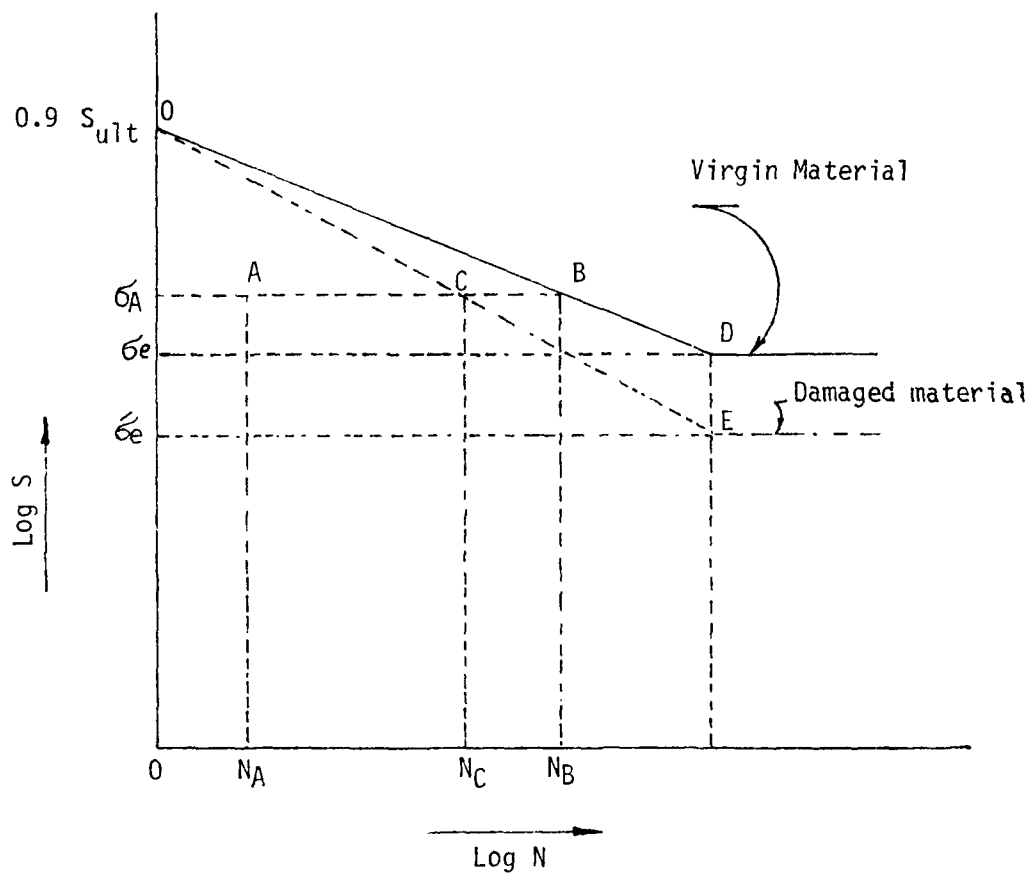


Figure 3. Cumulative Fatigue Damage

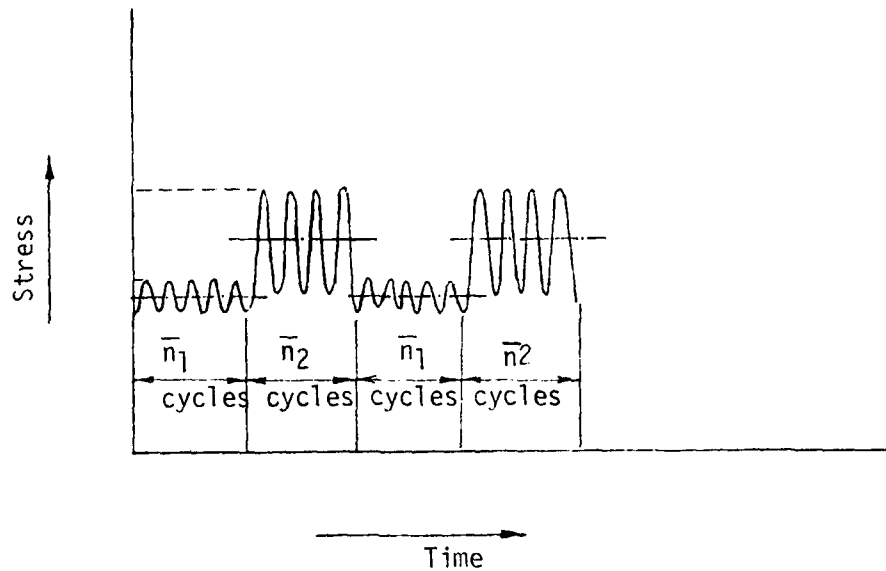


Figure 4. Two-Level Sinusoidal Stress History

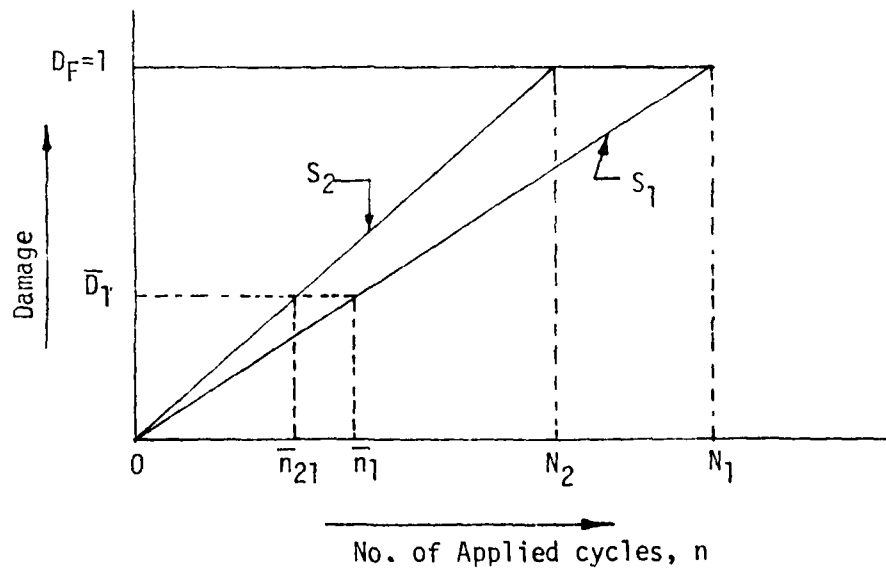


Figure 5. Damage Represented for Miner's Theory

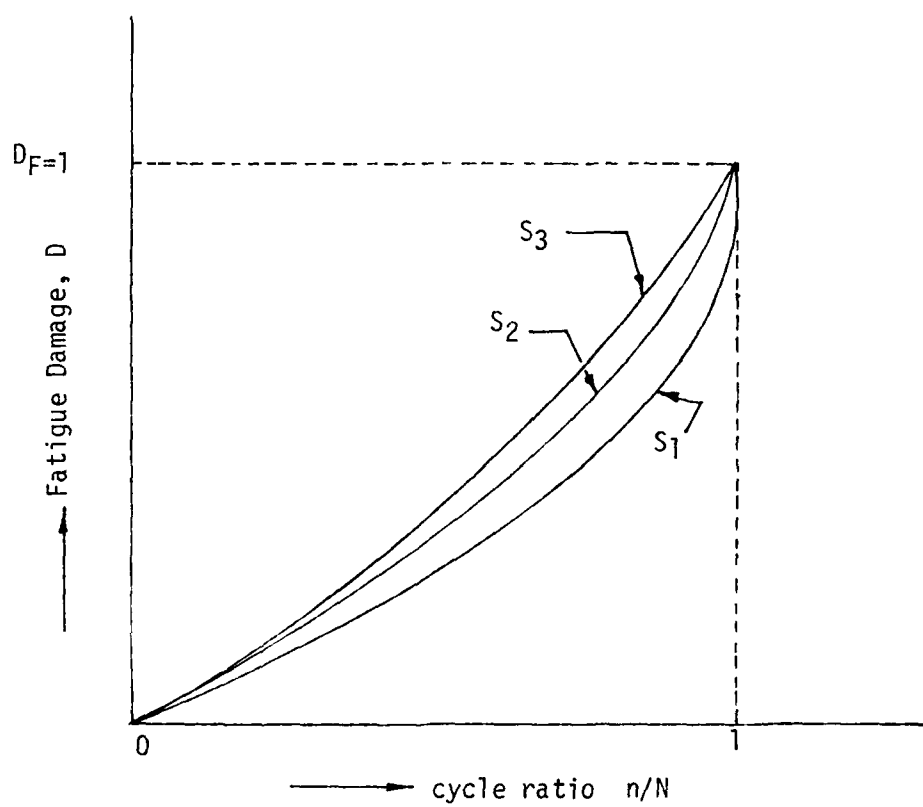


Figure 6. Stress Dependent Damage Representation

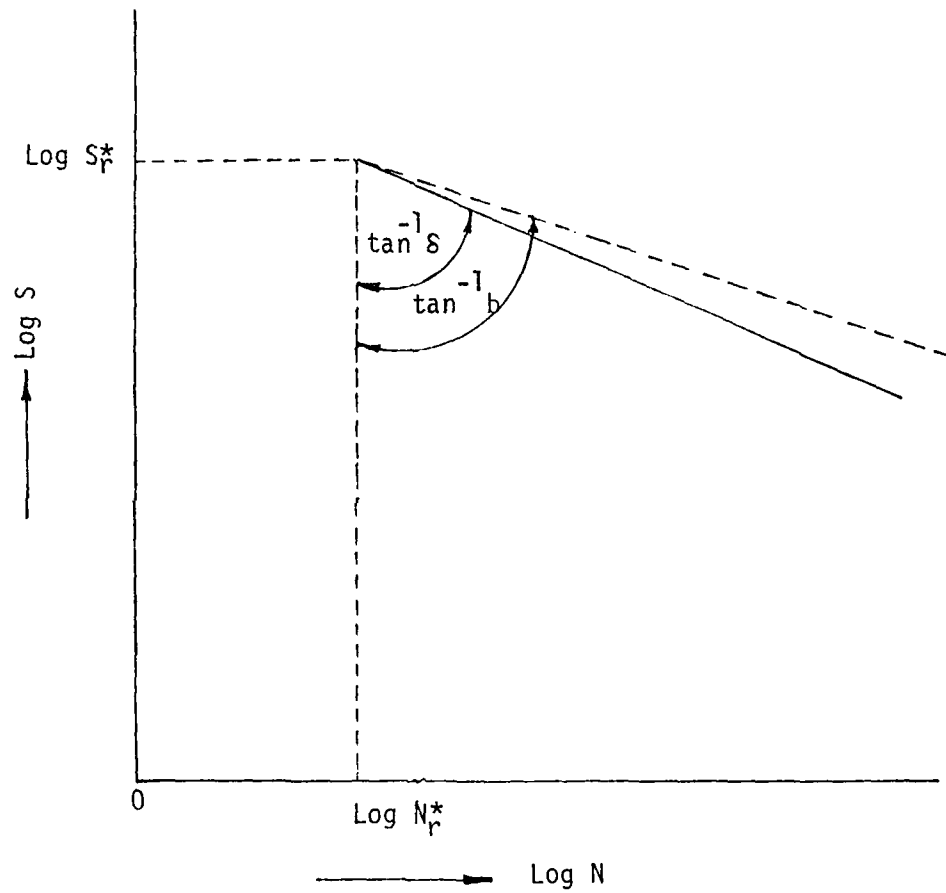


Figure 7. Modified S-N Diagram for Fruedenthal-Heller Theory

TABLE 1
ACTUAL EXPERIMENTAL STRESS SEQUENCE

Specimen No.	Stress Sequence			
	Stress in MPA/Nos. of Cycles in K-Cycles			
1	138/50	172/40	207/25	241/9.2 (F)
2	138/100	172/75	207/24 (F)	
3	138/25	172/30	207/24	276/6.6 (F)
4	276/5	241/10	207/24 (F)	
5	276/10	241/5	207/20	172/56 (F)
6	138/50	241/10	172/30	276/10 (F)
7	241/10	172/30	276/5	207/27 (F)

(F) Indicates failure.

TABLE 2

SPECIMEN	CUMULATIVE DAMAGE	
	KRAMER	COMPUTED
1	1.1	1.2961
2	1.4	1.5267
3	1.0	1.0623
4	1.1	1.0744
5	1.8	1.8779
6	1.1	1.1550
7	1.4	1.4366

TABLE 3

SEPCIMEN	NUMBER OF CYCLES TO FAILURE	
	EXPERIMENTAL	PREDICTED
1	9200	9000
2	24000	29000
3	6600	7000
4	24000	27200
5	56000	22200
6	10000	20000
7	27000	39000

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cumulative fatigue damage and fatigue life. The experimental values of fatigue damage and life are found to be in agreement with those predicted.

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